On contracting graphs to fixed pattern graphs

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joint work with Marcin Kamiński, Daniël Paulusma, Stefan Szeider and Dimitrios Thilikos

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Does H show up as a "pattern" in G?

Can we obtain H from G by

1. contracting edges and deleting edges?
2. contracting edges and deleting vertices?
3. contracting edges?

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   If so, then H is a minor of G.
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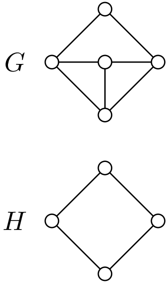

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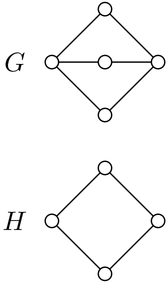

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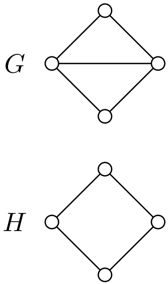

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   H-Contractibility problem

Computational complexity of recognizing pattern graphs

Can we obtain H from G

1. by contracting edges and deleting edges?
   H-MINOR CONTAINMENT problem

3. by contracting edges?
   H-CONTRACTIBILITY problem

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Theorem (Robertson & Seymour, 1995)

The H-Minor Containment problem is solvable in polynomial time for every fixed pattern graph H.

3. by contracting edges?
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Theorem (Brouwer & Veldman, 1987)

The P_4-Contractibility problem is NP*-complete.*

The H-Contractibility problem

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H-Contractibility *is* NP*-complete if* $H \in \{P_4, C_4\}$.

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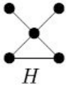
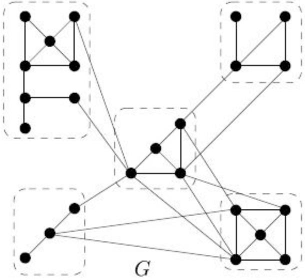
"The fact that H-Contractibility turns out to be NP-complete, even for such small graphs H as P_4 and C_4, makes us expect that the class of graphs H for which H-Contractibility is not NP-complete is very limited."

First, a bit more about contractibility

Graph G below is H-contractible.





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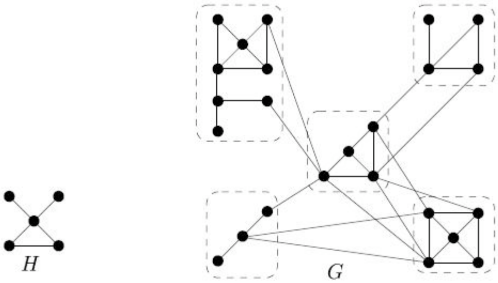
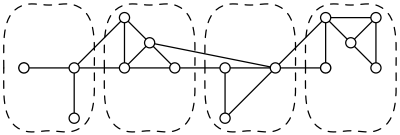


The dashed lines indicate the H-witness sets of G and together they form an H-witness structure of G.

A little bit more about contractibility

Two P_4-witness structures of a graph G:



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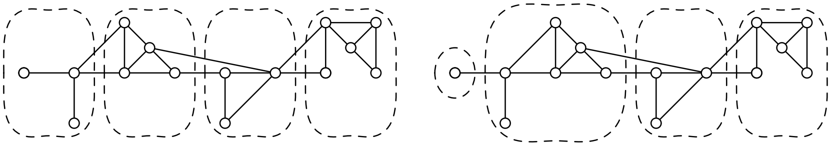

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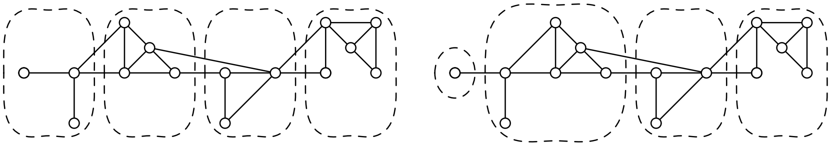

Lemma

Let $v \in V(H)$ be a vertex of degree 1. If G is H-contractible, then G has an H-witness structure $\mathcal{W}$ with $|W(v)| = 1$.

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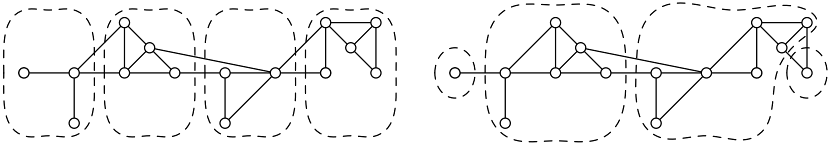

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Some families of graphs

P_1

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$$P_1 \cup P_1 \cup P_1$$

Some families of graphs

$3P_1$

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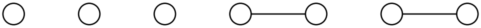
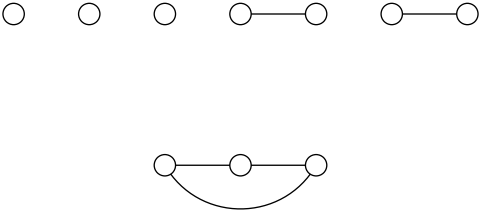


$3P_1 \cup 2P_2$

Some families of graphs



$K_3 \cup 3P_1 \cup 2P_2$

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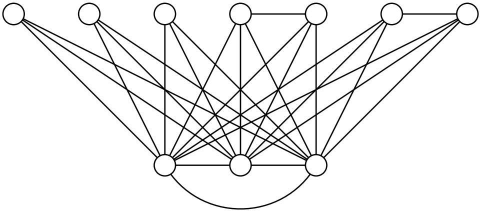

$$K_3 \bowtie (3P_1 \cup 2P_2)$$

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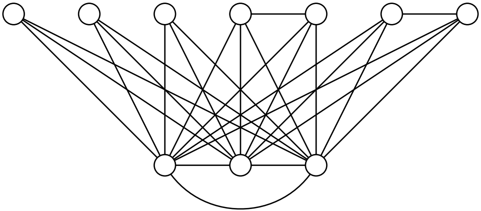
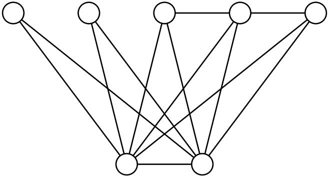


$H_3^*(3, 2)$

Some families of graphs



$H_2^*(2, 0, 1)$

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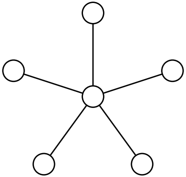

$H_1^*(5)$

Polynomially solvable cases of H-Contractibility

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The H-Contractibility problem is solvable in polynomial time if H is a star.

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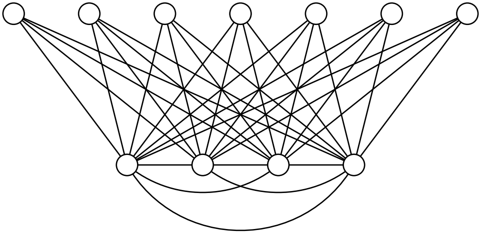
The H-Contractibility problem is solvable in polynomial time if $H = H_1^*(a_1, a_2, \ldots, a_k)$ or $H = H_2^*(a_1, a_2)$ or $H = H_3^*(a_1)$.

A new polynomially solvable case

Theorem

The H-Contractibility *problem is solvable in polynomial time if* $H = H_4^*(a_1)$.



$H_4^*(7)$

A closer look at previous work

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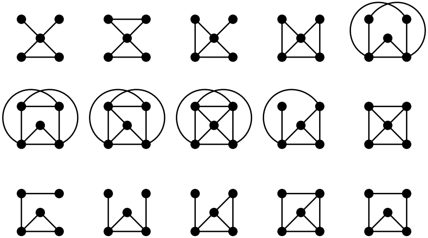

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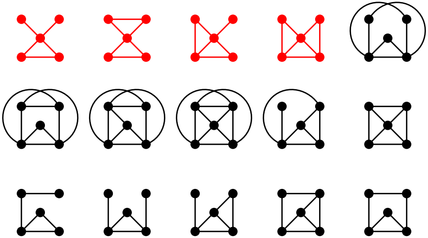

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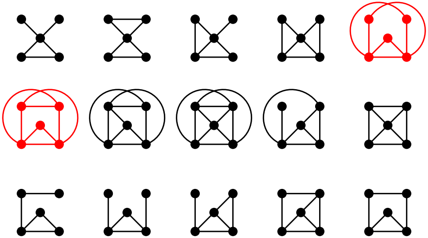

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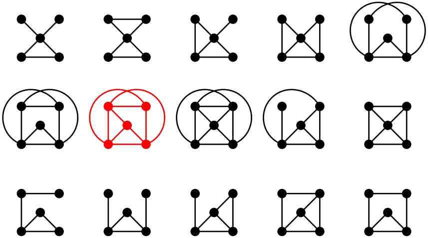

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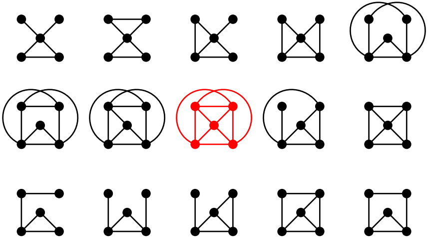

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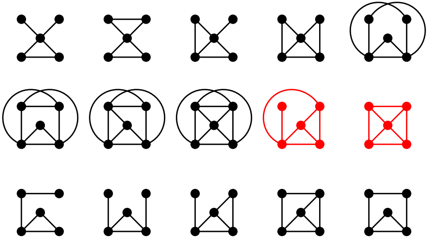

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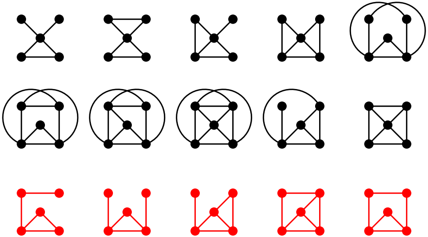

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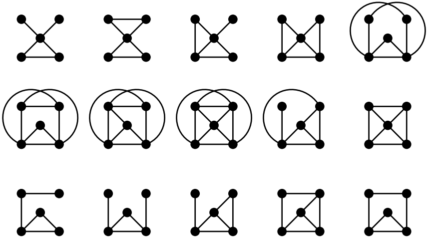

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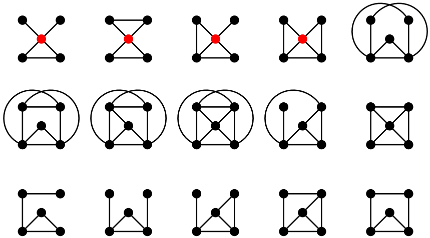

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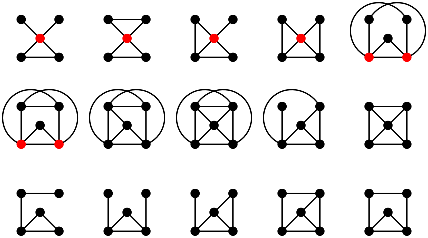

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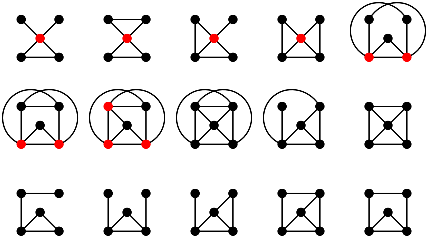

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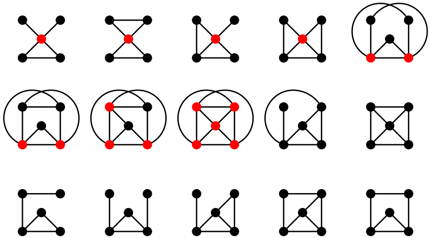

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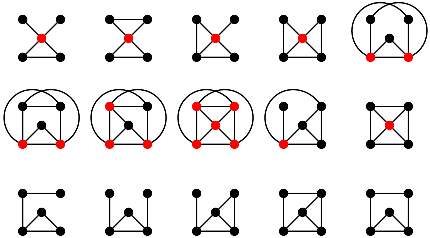

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Theorem (Levin, Paulusma & Woeginger, 2008)

Let H be a connected graph on at most 5 vertices.
The H-Contractibility problem is solvable in polynomial time if H has a dominating vertex, and is NP-complete otherwise.

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Is H-Contractibility in P iff H has a dominating vertex?

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Is H-Contractibility in P iff H has a dominating vertex? No!

Remember me?

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Computational complexity of recognizing pattern graphs

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The P_4-Contractibility problem is NP-complete.

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Theorem (Fellows et al., 2001)

The H^*-Induced Minor Containment problem is NP-complete.

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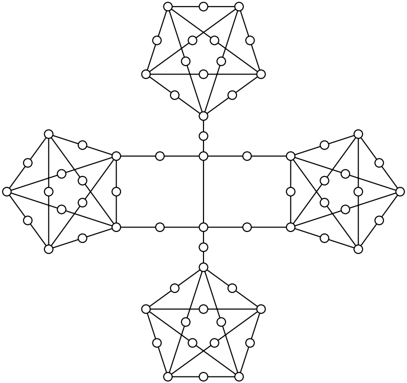
The graph H^*



Dominating vertex not a "classifier"

Three equivalent statements:

1. G has H as an induced minor
2. $K_1 \bowtie G$ has $K_1 \bowtie H$ as a contraction
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Dominating vertex not a "classifier"

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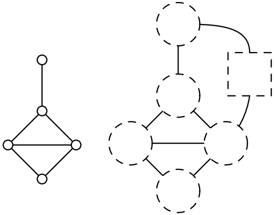

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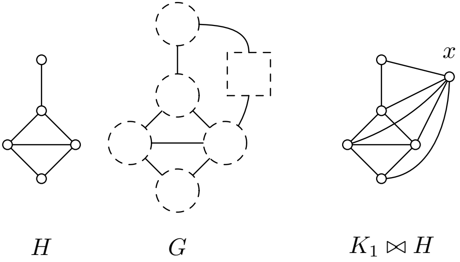

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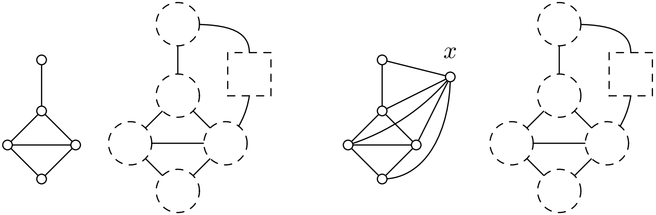

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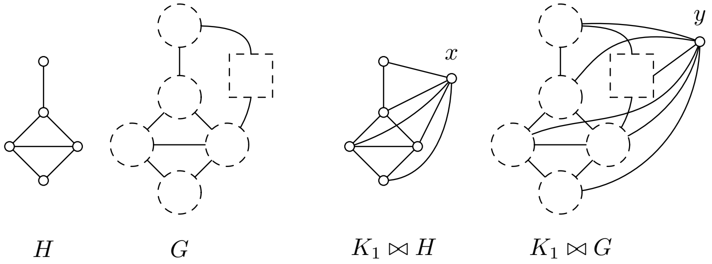

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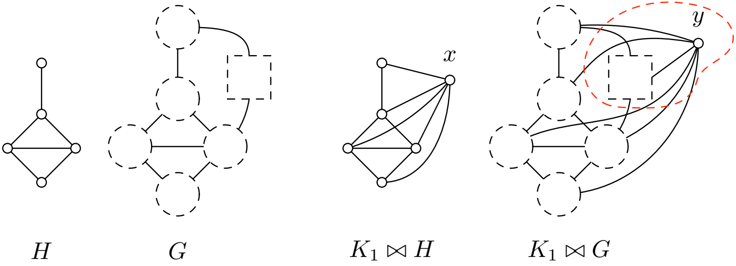

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Proposition

If H-Induced Minor Containment is NP-complete, then so are $(K_1 \bowtie H)$-Contractibility and $(K_1 \bowtie H)$-Induced Minor Containment.

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Corollary

For any $i \geq 1$, the $(K_i \bowtie H^*)$-Contractibility problem is NP-complete.

A new variant of the problem

(H, v)-Contractibility

Instance: Graph G, integer k.

Question: Does G have an H-witness structure $\mathcal{W}$ with $|W(v)| \geq k$?

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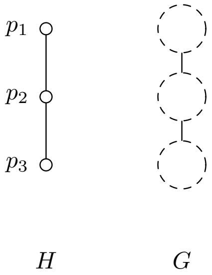
P_3-CONTRACTIBILITY



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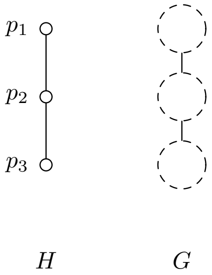

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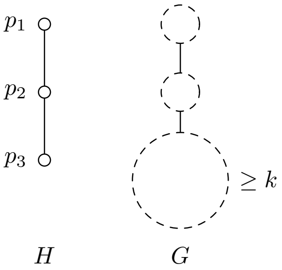

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Observation

If H-CONTRACTIBILITY in NP-complete, then so is (H, v)-CONTRACTIBILITY for any $v \in V(H)$.

Theorem

(P_3, p_2)-Contractibility *is solvable in polynomial time.* proof

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The (H, v)-CONTRACTIBILITY *problem is NP-complete if* H *is connected and* v *does not dominate* H*.*

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- (Bonus points) Give a computational complexity classification of the H-Contractibility problem.

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Thank you!

Pim van 't Hof
`www.dur.ac.uk/pim.vanthof`

(P_3, p_2)-Contractibility $\in P$

Lemma

Let $v \in V(H)$ be a vertex of degree 1. If G is H-contractible, then G has an H-witness structure $\mathcal{W}$ with $|W(v)| = 1$.

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Graph G is P_3-contractible if and only if G has two non-adjacent vertices u and v such that $G[V(G) \setminus \{u, v\}]$ is connected.

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Corollary

(P_3, p_2)-Contractibility *is solvable in polynomial time.*

back

(P_3, p_3)-CONTRACTIBILITY is NP-complete

Proof. Reduction from 3-SAT.

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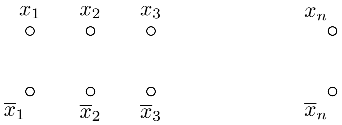

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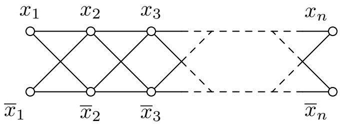

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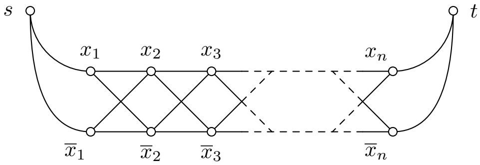

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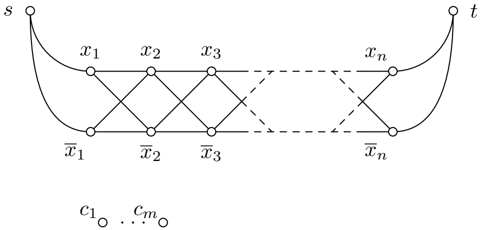

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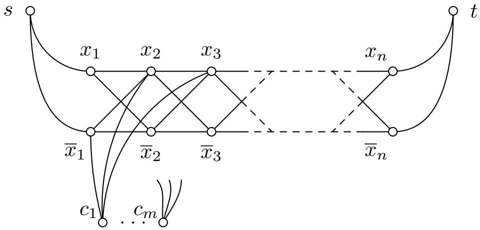

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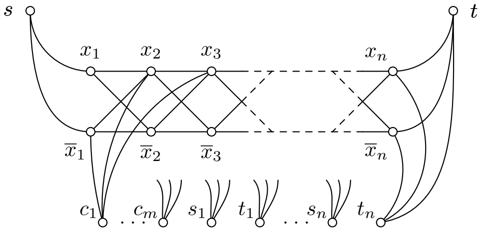

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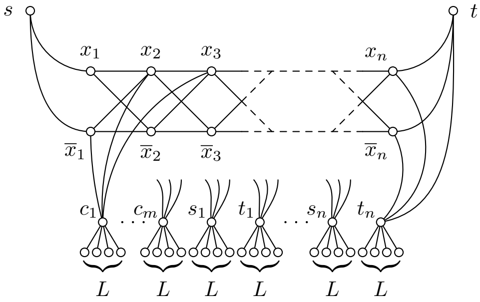

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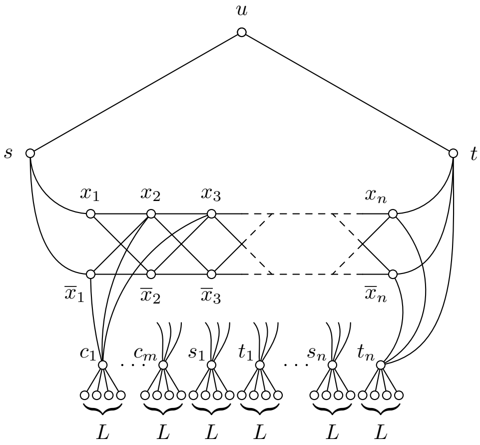

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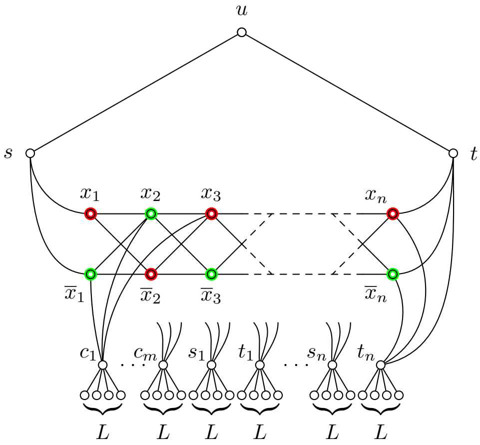

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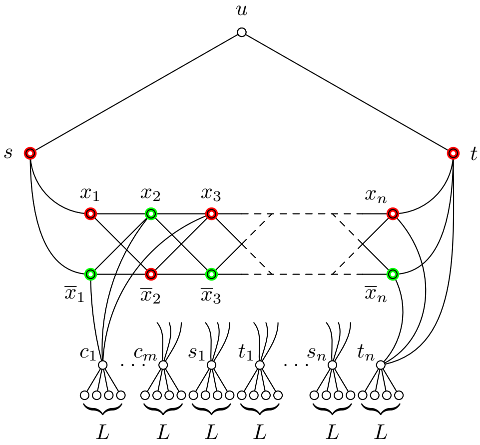

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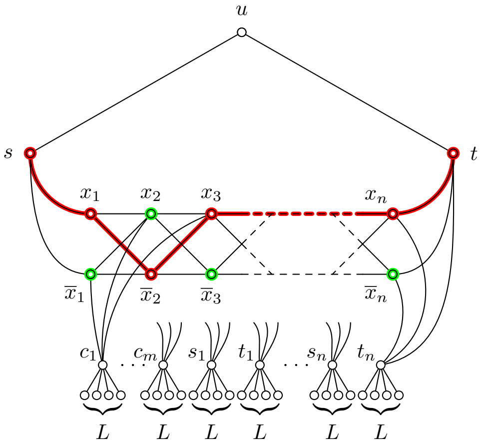

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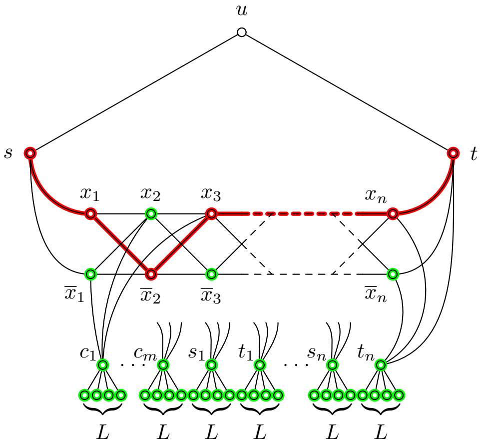

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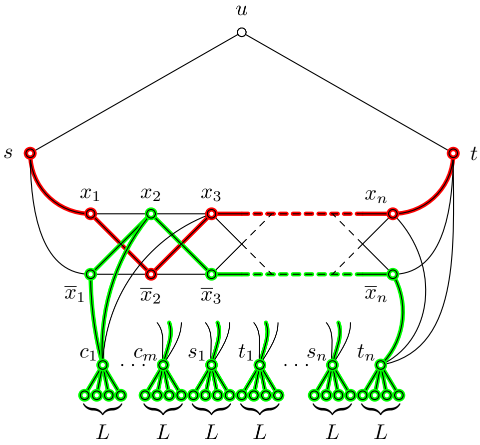

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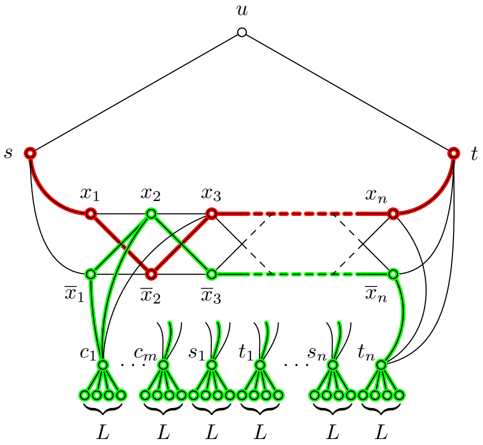

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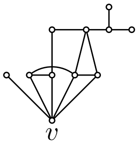

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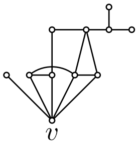

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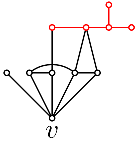

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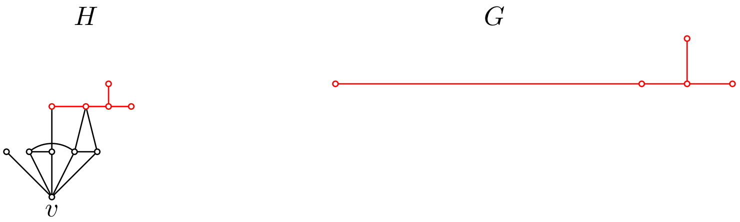

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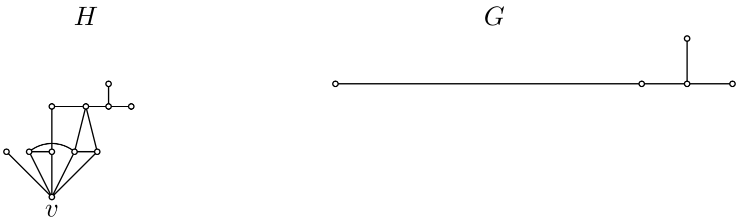

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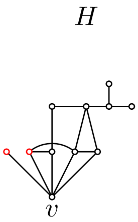

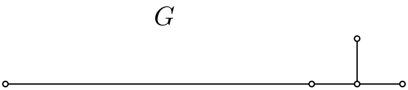

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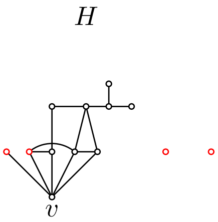

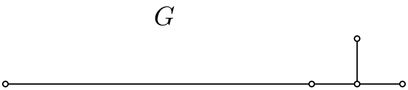

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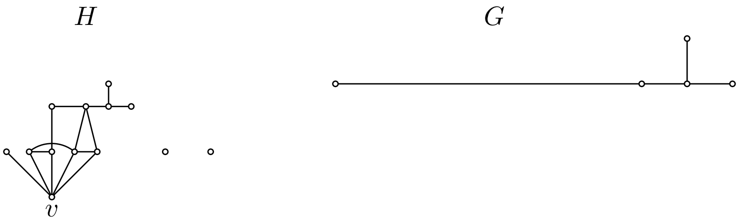

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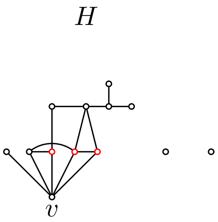

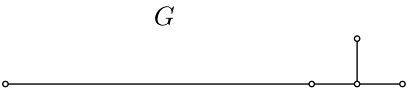

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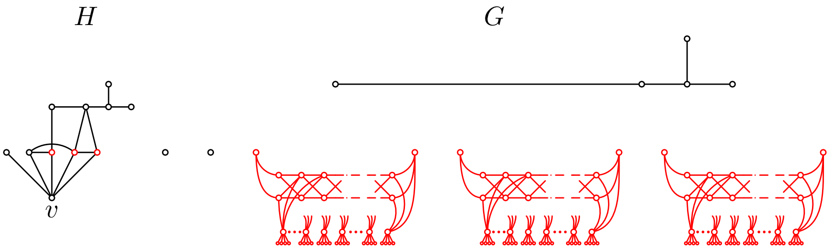

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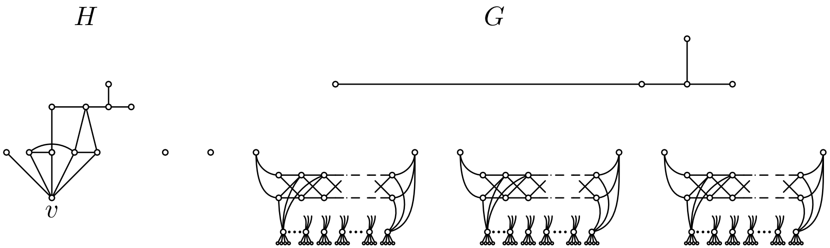

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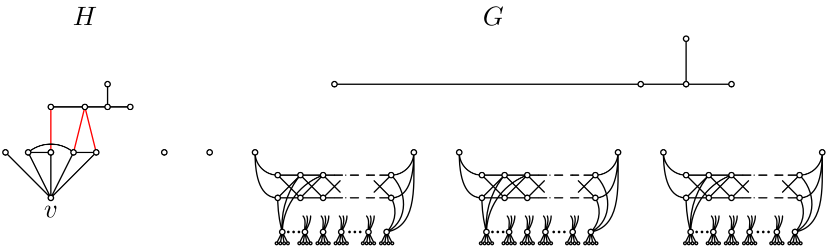

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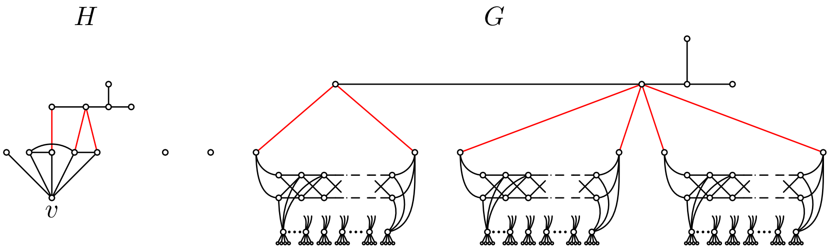

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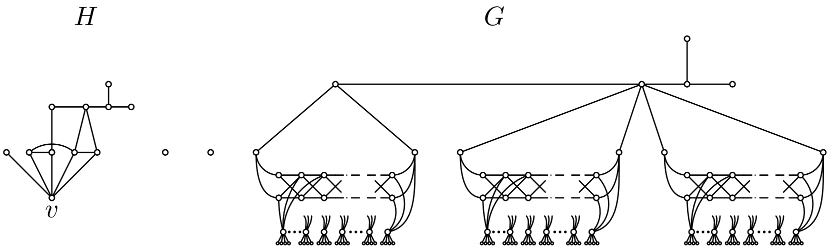

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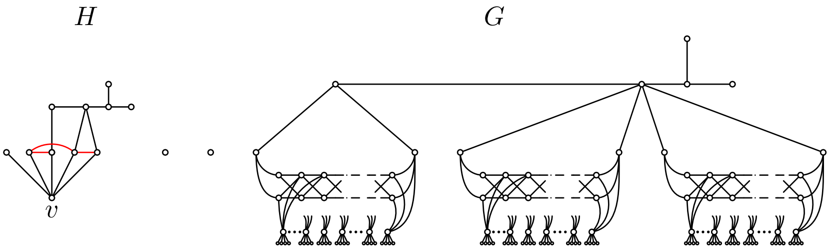

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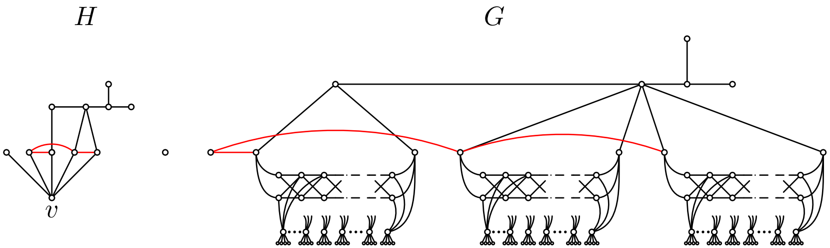

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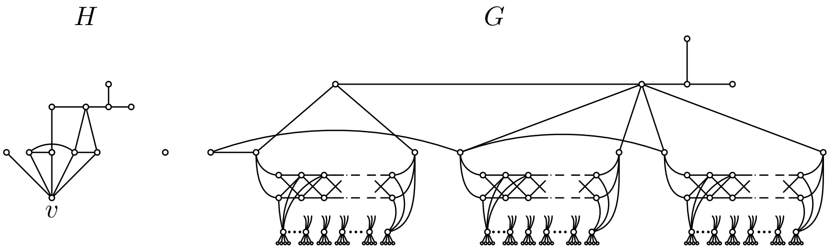

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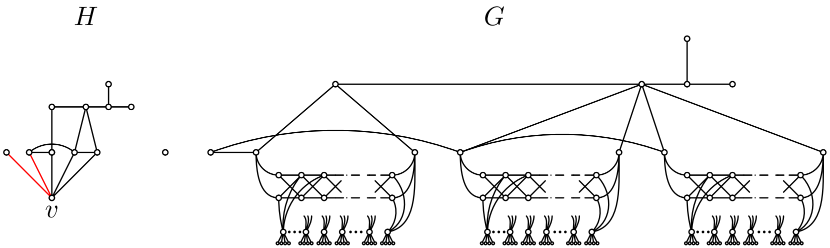

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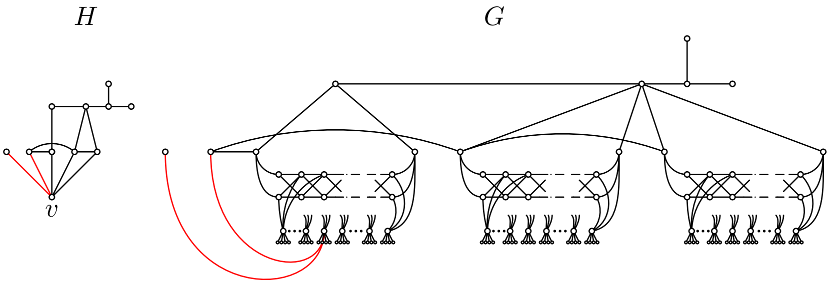

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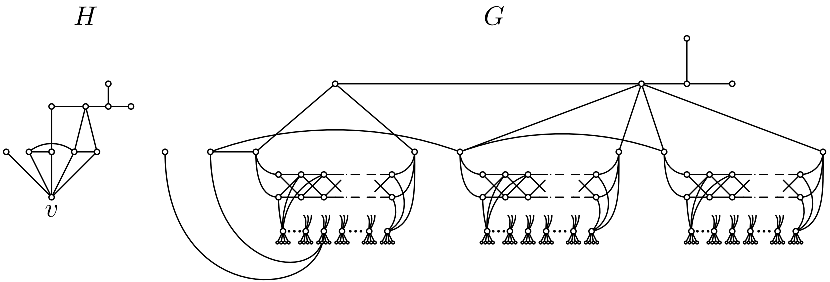

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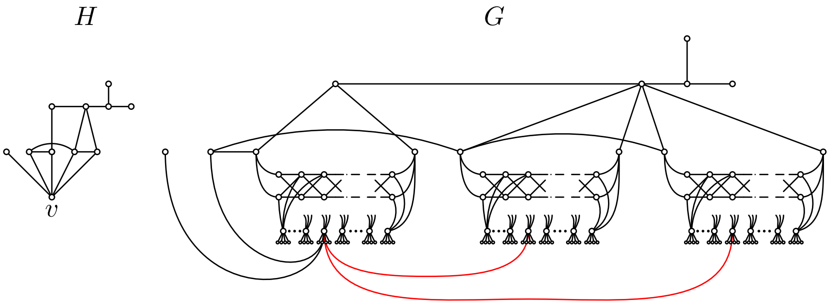

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◂ back